

You are the attacker against the CryptoNote protocol desiring to use the “multiple equations” approach to find a private key. Suppose you have:

- A secret key  $x$  (represented as an integer in the field  $G(2^{255-19})$  )
- A basepoint  $G$  on the elliptic curve  $E$ , which is for example Curve25519, which is a commonly used elliptic curve in Diffie-Helman cryptography<sup>1</sup>, and also happens to be the curve used in the CryptoNote protocol.
- A public key,  $P = xG$
- A “ring-image”  $I = xH_p(P)$ , where  $H_p(-)$  is a hash function<sup>2</sup> taking a point on  $E$  to another point on  $E$  (which happens to be another multiple of the basepoint  $G$ ).

and your goal is to solve, using the two equations  $P = xG$ , and  $I = xH_p(P)$  for the integer  $x$ . Note that  $H_p(P) = bG$  for some integer  $b$  (you may or may not know what that integer is, but let’s even assume you can control the integer  $b$  to make your job as attacker easier).

Thus you have the two equations

$$P = xG$$

and

$$I = xbG.$$

You know what the integer  $b$  is, and what the points  $G$ ,  $P$ , and  $I$  are, but not what the integer  $x$  is at this point.

Now compare this to the Elliptic Curve Diffie-Helman (ECDH) shared-key exchange. The ECDH procedure is as follows<sup>3</sup>:

- Profesor Xavier and Bob both know a basepoint  $G$  on the curve  $E$
- Profesor Xavier selects a secret integer  $x$  and computes the point  $P = xG$  on the curve  $E$
- Bob selects a secret integer  $b$  and computes the point  $B = bG$  on the curve  $E$

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<sup>1</sup><http://cr.yyp.to/ecdh/curve25519-20060209.pdf>

<sup>2</sup>Keccak1600 is used in CryptoNote, but the hash function doesn’t matter for the sake of this document

<sup>3</sup>Silverman, Arithmetic of Elliptic Curves pages 375-380

- Professor Xavier and Bob exchange values of  $P$  and  $B$  over an insecure communications line.
- Bob computes  $bP = bxG = I$  and Professor Xavier computes  $xB = xbG$  so they both know the shared key  $I = bxG$ .

Now the question is, can Bob, who knows the three pieces of information  $b$ ,  $bxG = I$ , and  $xG = P$  use these pieces of information to compute  $x$ , Xavier's private key? This is the exact same problem which is given above in the supposed "CryptoNote Attack." Because in Diffie-Helman it is well-known<sup>4</sup> to be difficult (computationally infeasible) to solve for the other party's private key (even in the case of multiple secret key uses - see section 3 in Bernstein's paper in the footnote), and since both problems involve exactly the same equations, that implies that no such attack, as outlined above, is possible against CryptoNote protocol. - Shen.Noether

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<sup>4</sup><http://cr.yyp.to/ecdh/curve25519-20060209.pdf>